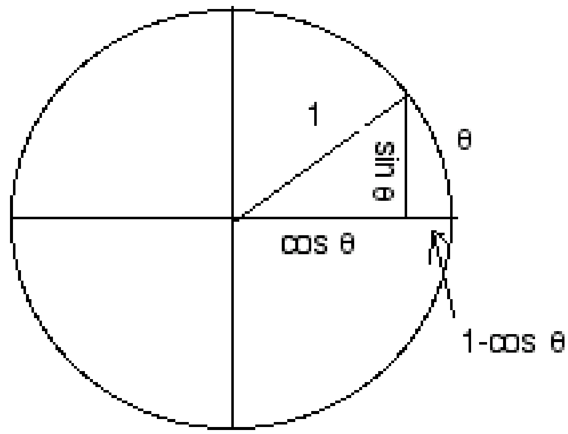
MATH 135 Lecture Notes
November 2, 2006

Sections 7.3 & 7.4
Derivatives of Trigonometric Functions

To find the derivative of the sine function we will need two limits:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x\to 0}\frac{1-\cos x}{x} = 0 .$$



In the figure at the right we see that sin θ and θ are nearly the same for small values of θ
On the other hand 1 – cos θ is very small compared to θ.

Further confirmation of these limits may be found by completing this table.
Let Y1 = sin(X)/X and Y2 = (1 – cos (X))/X
and select Ask for the independent variable under 2nd Tbl Set.

X	0.5	0.1	0.01	0.001	0.0001	0.00001
SIN (X)/X						
(1 –COS(X))/X						

We can now find rules for the derivatives of the six trigonometric functions. Note that we will us x as the argument of the function rather than θ.

1. $y = \sin x$. Apply the definition of derivative and the "delta" process

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x+\cos x\sin\Delta x-\sin x}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\sin x(\cos\Delta x-1)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\cos x\sin\Delta x}{\Delta x} = \sin x\cdot 0+\cos x\cdot 1 = \cos x .$$

Therefore $\frac{d(\sin x)}{dx} = \cos x$

2. $y = \cos x$. Use the complementary angle identity to rewrite the function as $y = \sin(\frac{\pi}{2}-x)$.

Use the chain rule with $u = \frac{\pi}{2}-x$ to obtain $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (\cos u)(-1) = -\cos(\frac{\pi}{2}-x) = -\sin x$.

Therefore $\frac{d(\cos x)}{dx} = -\sin x$

The derivatives of $y = \tan x$ and $y = \cot x$ may be obtained using the quotient rule and the Pythagorean Identity (page 244)

3. $\dfrac{d(\tan x)}{dx} = \sec^2 x$

4. $\dfrac{d(\cot x)}{dx} = -\csc^2 x$

The derivatives of y = sec x and y = csc x may be obtained using the power rule and the chain rule. (page 244)

5. $\dfrac{d(\sec x)}{dx} = \sec x \tan x$

6. $\dfrac{d(\csc x)}{dx} = -\csc x \cot x$

Learn these six rules in pairs, paying careful attention to signs (+ or –).

Using derivatives to analyze graphs.

1. The function $y = \sin 2x$ has a period of π and an amplitude of 1. Its derivative $\dfrac{dy}{dx}$ has a period of π and an amplitude of 2. You should be able to look at the graph of $y = \sin 2x$ and explain why.
2. Example 3 on page 240 shows the derivative of $y = \sin^2 x$. Use the double angle identities to show that the graph of this function has amplitude of ½ and a period of π.
3. Show that the function $y = \tan x$ is increasing wherever it is defined and that it has a point of inflection whenever the graph crosses the x-axis.
4. Show that y = sin x and $y = \cos x$ have the property that $f(x) = f''(x)$ for all x.

A related rates problem

Example 5 page 241. Think about why the function of power involves the *square* of the cosine function rather than just cosine to the first power. Also note the use of the double angle formula.

Exercises:

pages 242-243: 1, 5, 7, 13, 15, 19, 25, 37, 41, 43, 47
pages 246-247: 1, 5, 7, 21, 47